Inter (Part-I) 2018

Mathematics	Group-II	PAPER: I
Time: 2.30 Hours	(SUBJECTIVE TYPE)	Marks: 80

SECTION-I

2. Write short answers to any EIGHT (8) questions: (16)

(i) Does the set{1, –1} close w.r.t. :

(a) addition (b) multiplication

Ans It is closed w.r.t multiplication but not w.r.t addition.

$\otimes$	1	–1
1	1	–1
–1	–1	1

(ii) Find multiplicative inverse of the complex number (–4, 7).

Ans

$$M.I = \left[\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right]$$

Here $a = -4,\ b = 7$

$$M.I = \frac{-4}{(-4)^2+(7)^2}, \frac{-7}{(-4)^2+(7)^2}$$

$$= \frac{-4}{16+49}, \frac{-7}{16+49}$$

$$= \left[\frac{-4}{65}, \frac{-7}{65}\right]$$

(iii) If $z = 1 - i\sqrt{3}$, then find $|z|$.

Ans Let $z = 1 - i\sqrt{3}$

or

$$z = 1 + i(-\sqrt{3})$$

$$\therefore \quad |z| = \sqrt{(1)^2 + (-\sqrt{3})^2}$$

$$= \sqrt{1+3}$$

$$= \sqrt{4}$$

$$= 2$$

(iv) Write inverse and contrapositive of $q \to p$.

Ans

Inverse $= \sim q \to \sim p$

Contrapositive $= \sim p \to \sim q$

(v) If $A = \{a, b, c\}$, then write all subsets of A and find P(A).

Ans

$$A = \{a, b, c\}$$

$$P(A) = \phi\ \{a\}\ \{b\}\ \{c\}\ \{a, b\}\ \{b, c\}\ \{a, c\}, \{a, b, c\}$$

(vi) Show that set of natural number is not a group w.r.t. addition.

Ans **1. Closure property:**

Satisfied *i.e.,* $\forall\ a, b, \in N,\ a + b \in N$

2. Associativity:

Satisfied *i.e.,* $\forall\ a, b, c \in N,\ a + (b + c) = (a + b) + c$

3. Identity property:

Identity of any number not exists.

4. Inverse property:

Inverse of any number not exists.

(vii) Define diagonal matrix with an example.

Ans "Let $A = [a_{ij}]$ be a square matrix of order n. If $a_{ij} = 0$ for all $i \neq j$ and at least one $a_{ij} = 0$ for $i = j$, that is, some elements of the principal diagonal of A may be zero but not all, then matrix A is called a diagonal matrix. *e.g.,*

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}_{3 \times 3}$$

(viii) If $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$, then find A^{-1}.

Ans Let $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 3 \end{vmatrix}$$

$$= 2(3) - 6(1)$$

$$= 6 - 6$$

$$= 0$$

The further solution does not exist.

(ix) Without expansion show that $\begin{vmatrix} 6 & 7 & 8 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix} = 0$.

Ans L.H.S :

Add C_3 in C_1

$$= \begin{vmatrix} 14 & 7 & 8 \\ 8 & 4 & 5 \\ 6 & 3 & 4 \end{vmatrix}$$

Taking '2' common from C_1

$$= (2)\begin{vmatrix} 7 & 7 & 8 \\ 4 & 4 & 5 \\ 3 & 3 & 4 \end{vmatrix}$$

As $C_1 = C_2$, so matrix becomes zero

$$= 2 \times 0$$

$$= 0 = \text{R.H.S.}$$

(x) Find four 4th roots of unity.

Ans Let

$$x^4 = 1$$

$$x^4 - 1 = 0$$

$$(x^2 + 1)(x^2 - 1) = 0$$

$x^2 + 1 = 0$	$x^2 - 1 = 0$
$x^2 = -1$	$x^2 = 1$
$\sqrt{x^2} = \sqrt{-1}$	$\sqrt{x^2} = \sqrt{1}$
$x = \pm i$	$x = \pm 1$

roots, $-1, 1, i, -i$

(xi) If α, β are roots of $x^2 - px - p - c = 0$, show that $(1 - \alpha)(1 + \beta) = 1 - c$.

Ans

$$x^2 - px - p - c = 0$$

Here $a = 1, b = -p, c' = -p - c$

$$\text{Sum of roots} = \alpha + \beta = \frac{-b}{a} = \frac{-(-p)}{1} = p$$

$$\text{Product of roots} = \alpha\beta = \frac{c'}{a} = \frac{-p-c}{1} = -p - c$$

$$\text{L.H.S} = (1 + \alpha)(1 + \beta)$$

$$= 1 + 1\alpha + 1\beta + \alpha\beta$$

$$= 1 + (\alpha + \beta) + (\alpha\beta)$$

$$= 1 + p - p - c$$

$$= 1 - c = \text{R.H.S}$$

(xii) Find quadratic equation whose roots are $2\omega, 2\omega^2$, where ω is cube roots of unity.

Ans Sum of roots $= 2\omega + 2\omega^2$

$$= 2(\omega + \omega^2)$$
$$= 2(-1) = -2$$

Product of roots $= 2\omega \cdot 2\omega^2$
$$= 4\omega^3$$
$$= 4(1) = 4$$

So required equation,
$$x^2 - (\text{sum of roots})\, x + \text{product of roots} = 0$$
$$x^2 - (-2)x + 4 = 0$$
$$x^2 + 2x + 4 = 0$$

3. Write short answers to any EIGHT (8) questions: (16)

(i) Resolve $\dfrac{x^2+1}{(x+1)(x-1)}$ into partial fractions.

Ans
$$\frac{x^2+1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} \qquad \text{(i)}$$

Multiply equation (i) with $(x+1)(x-1)$, we get
$$x^2 + 1 = A(x-1) + B(x+1) \qquad \text{(ii)}$$

Put $x + 1 = 0 \Rightarrow x = -1$ in eq. (ii),
$$(-1)^2 + 1 = A(-1-1) + B(-1+1)$$
$$1 + 1 = A(-2) + B(0)$$
$$2 = -2A \Rightarrow \boxed{A = -1}$$

Put $x - 1 = 0 \Rightarrow x = 1$ in eq. (ii),
$$(1)^2 + 1 = A(1-1) + B(1+1)$$
$$2 = A(0) + B(2) \Rightarrow \boxed{B = 1}$$

Put value of A and B in eq. (i),
$$\frac{x^2+1}{(x+1)(x-1)} = \frac{-1}{x+1} + \frac{1}{x-1}$$

(ii) Find the indicated term of the sequence 2, 6, 11, 17, ---- $a_7 = ?$

Ans
$$a_1 = 2$$
$$a_2 = 6$$
$$a_3 = 11$$
$$a_n = 17$$
$$d = a_2 - a_1 = 4$$
$$d = a_3 - a_2 = 5$$

$$d = a_4 - a_3 = 6$$

Similarly,

$$a_5 = a_4 + 7 = 17 + 7$$
$$= 24$$
$$a_6 = a_5 + 8$$
$$= 24 + 8$$
$$a_6 = 32$$
$$a_7 = a_6 + 9$$
$$= 32 + 9$$
$$a_7 = 41$$

(iii) Sum the series up to n-terms $\frac{1}{1-\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1+\sqrt{x}} + \text{-----}$.

Ans Here $a_1 = \frac{1}{1-\sqrt{x}}$

$$d = \frac{1}{1-x} - \frac{1}{1-\sqrt{x}}$$
$$= \frac{(1-\sqrt{x})-(1-x)}{(1-x)(1-\sqrt{x})}$$
$$= \frac{x-\sqrt{x}}{(1-x)(1-\sqrt{x})}$$

$$\text{Sum} = S_n = \frac{n}{2}\{2a + (n-1)d\}$$
$$= \frac{n}{2}\left\{2\left(\frac{1}{1-\sqrt{x}}\right) + (n-1)\frac{x-\sqrt{x}}{(1-x)(1-\sqrt{x})}\right\}$$
$$= \frac{n}{2}\left\{\frac{2}{1-\sqrt{x}} + \frac{(n-1)(x-\sqrt{x})}{(1-x)(1-\sqrt{x})}\right\}$$
$$= \frac{n}{2}\left\{\frac{2(1-x) + n(x-\sqrt{x}) - 1(x-\sqrt{x})}{(1-x)(1-\sqrt{x})}\right\}$$
$$= \frac{n}{2}\left\{\frac{2-2x+nx-n\sqrt{x}-1x+\sqrt{x}}{(1-x)(1-\sqrt{x})}\right\}$$
$$= \frac{n}{2}\left\{\frac{2-3x+nx-n\sqrt{x}+\sqrt{x}}{(1-x)(1-\sqrt{x})}\right\}$$

(iv) Insert two G.Ms between 1 and 8.

Ans Let, G_1, G_2 be the two geometric means (G.M's) between 1 and 8.

So,

$$1, G_1, G_2, 8 \text{ are in G.P}$$

Here, $a = 1$,

$n = 4$,

$a_4 = 8$

We know that

$$a_n = ar^{n-1}$$

For $n = 4$

$$a_4 = ar^{4-1}$$

$$8 = 1(r)^3$$

$$(2)^3 = (r)^3$$

$\Rightarrow$ $r = 2$

Therefore,

$$G_1 = ar = (1)(2) = 2$$

$$G_2 = ar^2 = (1)(2)^2 = 4$$

So, the two G.M's between 1 and 8 are : 2, 4.

(v) **Find the sum of the infinite geometric series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \text{-----}$.**

Ans Here $a = \frac{1}{2}$, $a_1 = \frac{1}{4}$

$$r = \frac{a_1}{a} = \frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{4} \cdot 2 = \frac{1}{2}$$

Sum of infinite geometric series $= s_\infty = \frac{a}{1-r}$

$$= \frac{\left(\frac{1}{2}\right)}{1 - \frac{1}{2}}$$

$$= \frac{\frac{1}{2}}{\frac{2-1}{2}} = \frac{1}{2} \cdot \frac{2}{1}$$

$$S_\infty = 1$$

(vi) Find the 12^{th} term of the harmonic sequence $\frac{1}{3}, \frac{2}{9}, \frac{1}{6}$,------.

Ans $\frac{1}{3}, \frac{2}{9}, \frac{1}{6}$, ------ is an harmonic progression (H.P).

$\Rightarrow$ $3, \frac{9}{2}, 6$, --- is an average progression (A.P)

Here $a = 3$, $d = \frac{9}{2} - 3$

$$d = \frac{9-6}{2}$$

$$d = \frac{3}{2}$$

We know

$$a_n = a + (n-1)d$$

For

$$n = 12$$

$$a_{12} = a + (12-1)d$$

$$a_{12} = 3 + 11\left(\frac{3}{2}\right)$$

$$= 3 + \frac{33}{2}$$

$$= \frac{6+33}{2}$$

$$= \frac{39}{2}$$

$$a_{12} = \frac{39}{2}$$

is in average progression (A.P).

Thus the required term is $\frac{2}{39}$ in H.P because it is the reciprocal.

(vii) Evaluate $\frac{15!}{15!(15-15)!}$.

Ans $\frac{15!}{15!(0)!} = \frac{15!}{15! \times 1} = \frac{1}{1 \times 1} = 1 \qquad (\because \; 0! = 1)$

(viii) Find the value of n, when $\frac{12 \times 11}{2!} = {}^nC_{10}$.

Ans

$$
\begin{aligned}
{}^nC_{10} &= \frac{12 \times 11}{2\,!} \\
&= \frac{12 \times 11 \times 10!}{2! \times 10!} \\
&= \frac{12!}{2!(12-2)!}
\end{aligned}
$$

So $\quad {}^nC_{10} = {}^{12}C_2$

$${}^nC_{n-10} = {}^{12}C_2 \Rightarrow \quad n - 10 = 2$$

$$n = 12$$

(ix) There are 5 green and 3 red balls in a box, one ball is taken out, find the probability that the ball drawn is green.

Ans Total number of balls = 5 + 3 = 8

Total possible outcomes = ${}^8C_1 = 8$

Favourable outcomes = ${}^5C_1 = 5$

$$\text{Probability P} = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{5}{8}$$

(x) Find the number of the diagonals of a 6-sided figure.

Ans Number of diagonals

$$= {}^6C_2 - 6$$

$$= 15 - 6 = 9$$

(xi) Find the term involving x^4 in the expansion of $(3 - 2x)^7$.

Ans Let T_{r+1} be the required. Then

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$= \binom{7}{r} 3^{7-r} (-2x)^r$$

$$= \binom{7}{r} 3^{7-r} (-2)^r (x)^r \qquad \text{(i)}$$

For the term involving x^4, put exponent of x equal to 4, *i.e.*, $r = 4$

$$T_{4+1} = \binom{7}{4} 3^{7-4} (-2)^4 x^4$$

$$T_5 = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} (3^3)(16)\, x^4$$

$$= 15120\, x^4$$

(xii) Using binomial theorem find the value of $(1.03)^{1/3}$ up to three decimal places.

Ans $(1.03)^{1/3} = (1 + 0.03)^{1/3}$

$$= 1 + \frac{1}{3}(0.03) + \frac{\frac{1}{3}\left(\frac{1}{3} - 1\right)}{2!}(0.03)^2 + \text{---}$$

$$= 1 + 0.01 - \frac{1}{9}(0.009) + \text{---}$$

$$= 1 + 0.01 - 0.0001 + \text{---}$$

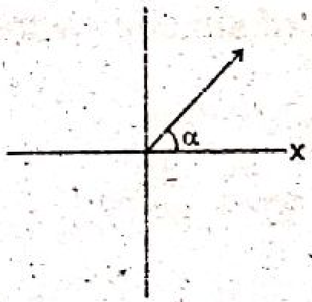
$$= 1.0099$$

4. Write short answers to any NINE (9) questions: (18)

(i) Define angle in the standard position with figure.

Ans An angle is said to be in standard position if its vertex lies at the origin of a rectangular coordinate system and its initial side along the positive x-axis.

For example,



(ii) Find x, if $\tan^2 45° - \cos^2 60° = x \sin 45° \cos 45° \tan 60°$.

Ans $\tan^2 45° - \cos^2 60° = x \sin 45° \cos 45° \tan 60°$

$$(1) - \left(\frac{1}{2}\right)^2 = x\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(\sqrt{3})$$

$$1 - \frac{1}{4} = x\frac{\sqrt{3}}{2}$$

$$\frac{3}{4} = \frac{x\sqrt{3}}{2}$$

$$\frac{3}{4} \times \frac{2}{\sqrt{3}} = x$$

$$\frac{\sqrt{3}}{2} = x$$

$$\Rightarrow \quad x = \frac{\sqrt{3}}{2}$$

(iii) Prove that $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$.

Ans $L.H.S = \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

$$= \frac{1(1-\sin\theta) + 1(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{1-\sin\theta+1+\sin\theta}{1-\sin^2\theta}$$

As $\quad \sin^2\theta + \cos^2\theta = 1$

$$\cos^2\theta = 1 - \sin^2\theta$$

So, $L.H.S = \frac{2}{\cos^2\theta}$

$$= 2 \cdot \frac{1}{\cos^2\theta}$$

$$= 2\sec^2\theta$$

$= R.H.S$

(iv) Find the value of sin 540° without using calculator.

Ans $\sin 540^\circ$

$= \sin(540^\circ + 0)^\circ$

$= \sin(6 \times 90 + 0)^\circ$

$= 6\sin 90^\circ + \sin 0^\circ$

$= 6(0) + 0$

$= 0$

(v) Prove that $\tan\left(\frac{\pi}{4} - \theta\right) + \tan\left(\frac{3\pi}{4} + \theta\right) = 0$.

Ans $L.H.S = \tan\left(\frac{\pi}{4} - \theta\right) + \tan\left(\frac{3\pi}{4} + \theta\right)$

$$= \frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta} + \frac{\tan\frac{3\pi}{4} + \tan\theta}{1 - \tan\frac{3\pi}{4}\tan\theta}$$

$$= \frac{1 - \tan\theta}{1 + \tan\theta} + \frac{-1 + \tan\theta}{1 + \tan\theta}$$

$$= \frac{1 - \tan\theta - 1 + \tan\theta}{1 + \tan\theta}$$

$$= \frac{0}{1 + \tan\theta} = 0 = \text{R.H.S}$$

Hence proved

L.H.S = R.H.S

(vi) Express $\sin(x + 45^\circ)\sin(x - 45^\circ)$ as sum or difference.

Ans $\sin(x + 45^\circ)\sin(x - 45^\circ)$

$$= \frac{-1}{2}[-2\sin(x + 45^\circ)\sin(x - 45^\circ)]$$

$$= \frac{-1}{2}[\cos(x + 45^\circ + x - 45^\circ) - \cos(x + 45^\circ - x + 45^\circ)]$$

$$= \frac{-1}{2}[\cos 2x - \cos 90^\circ]$$

$$= \frac{-1}{2}(-1)[\cos 90^\circ - \cos 2x] \Rightarrow \frac{1}{2}[\cos 90^\circ - \cos 2x]$$

(vii) Find the period of $\cos\frac{x}{6}$.

Ans $\cos\frac{x}{6} = \cos\left[\frac{x}{6} + 2\pi\right]$

$$= \cos\left[\frac{x + 12\pi}{6}\right]$$

$$= \frac{1}{6}\cos(x + 12\pi)$$

Period of $\cos\frac{x}{6}$ is 12π.

(viii) Find the area of triangle ΔABC, in which $b = 37$, $c = 45$ and $\alpha = 30^\circ 50'$.

Ans $b = 37$, $c = 45$, $\alpha = 30^\circ 50'$

$$\Delta = \frac{1}{2} b \cdot c \sin\alpha$$

$$= \frac{1}{2} \times 37 \times 45 \sin 30°50'$$

$$= 426.69 \text{ sq. units}$$

(ix) Prove that $r\, r_1 r_2 r_3 = \Delta^2$ (Using usual notation).

Ans $r\, r_1\, r_2\, r_3 = \frac{\Delta}{s} \times \frac{\Delta}{s-a} \times \frac{\Delta}{s-b} \times \frac{\Delta}{s-c}$

$$= \frac{\Delta^4}{s(s-a)(s-b)(s-c)}$$

$$= \frac{\Delta^4}{\Delta^2}$$

$$= \Delta^2 \quad \text{Proved.}$$

(x) Prove that $(r_1 + r_2) \tan\frac{\gamma}{2} = c$ (Using usual notation).

Ans Given,

$$(r_1 + r_2) \tan\frac{\gamma}{2} = c$$

By taking,

$$\text{L.H.S} = (r_1 + r_2) \tan\left(\frac{\gamma}{2}\right)$$

$$= \left(\frac{\Delta}{s-a} + \frac{\Delta}{s-b}\right) \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$= \Delta\left(\frac{1}{s-a} + \frac{1}{s-b}\right) \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$= \Delta\left(\frac{1(s-b) + 1(s-a)}{(s-a)(s-b)}\right) \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$= \Delta\left(\frac{s-b+s-a}{(s-a)(s-b)}\right) \frac{\sqrt{(s-a)(s-b)}}{\sqrt{s(s-c)}}$$

$$= \Delta\left(\frac{2s-a-b}{(s-a)(s-b)}\right) \frac{\sqrt{(s-a)\,(s-b)}}{\sqrt{s(s-c)}} \times \frac{\sqrt{(s-a)(s-b)}}{\sqrt{(s-a)(s-b)}}$$

$$= \Delta\left(\frac{2s-(a+b)}{(s-a)(s-b)}\right) \frac{\sqrt{(s-a)^2\,(s-b)^2}}{s(s-a)(s-b)(s-c)}$$

$$= \frac{\Delta\,[2s-(a+b)]}{(s-a)(s-b)} \frac{(s-a)(s-b)}{\Delta}$$

$$\left[\because \sqrt{s(s-a)(s-b)(s-c)} = \Delta\right]$$

As

$$2s = a + b + c$$
$$= a + b + c - a - b$$
$$= c$$
$$= R.H.S$$

(xi) Find domain and range of $y = \cos^{-1} x$.

Ans

$$y = \cos^{-1} x$$
$$x = \cos y$$

Here 'y' is the angle whose cosine is 'x'

$$\text{Domain} = -1 \leq x \leq 1$$
$$\text{Range} = 0 \leq y \leq \pi$$

(xii) Solve the equation $\sin x = \frac{1}{2}$.

Ans

$$\sin x = \frac{1}{2}$$

As sin x is positive in I and II quadrant with reference angle $x = \frac{\pi}{6} = 30^{\circ}$.

So, $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ where $x \in [0, 2\pi]$

$$x = \frac{\pi}{6} + 2n\pi \quad \text{and} \quad \frac{5\pi}{6} + 2n\pi$$

(xiii) Find solutions of $\cot\theta = \frac{1}{\sqrt{3}}$ which lie in $[0, 2\pi]$.

Ans

$$\cot\theta = \frac{1}{\sqrt{3}}$$
$$\tan\theta = \sqrt{3}$$
$$\theta = \tan(\sqrt{3})$$
$$\theta = \frac{\pi}{3}$$
$$\text{Angles} = \frac{\pi}{3}, \pi + \frac{\pi}{3}$$
$$\text{Angles} = \frac{\pi}{3}, \frac{4\pi}{3}$$

SECTION-II

NOTE: Attempt any Three (3) questions.

Q.5.(a) Convert the following theorem to logical form and prove it by constructing truth table: (5)

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Ans Constructing truth table

p	q	r	$q \wedge r$	$p \vee r$	$p \vee q$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	F	F
F	T	F	F	F	T	F	F
F	F	T	F	T	F	F	F
F	F	F	F	F	F	F	F

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$$

Thus $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) Solve the following system by reducing their augmented matrices to the echelon form: (5)

$$x + 2y + z = 2$$
$$2x + y + 2z = -1$$
$$2x + 3y - z = 9$$

Ans The augmented matrix is:

$$\begin{bmatrix} 1 & 2 & 1 & : & 2 \\ 2 & 1 & 2 & : & -1 \\ 2 & 3 & -1 & : & 9 \end{bmatrix}$$

Apply row operations $R_2 + (-2) R_1$; $R_3 + (-2) R_1$

$$\begin{bmatrix} 1 & 2 & 1 & : & 2 \\ 0 & -3 & 0 & : & -5 \\ 0 & -1 & -3 & : & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & : & 2 \\ 0 & 1 & 0 & : & 5/3 \\ 0 & -1 & -3 & : & 5 \end{bmatrix} \quad \left(\frac{-1}{3}\right) R_2$$

$$\begin{bmatrix} 1 & 2 & 1 & : & 2 \\ 0 & 1 & 0 & : & 5/3 \\ 0 & 0 & -3 & : & 20/3 \end{bmatrix} \quad R_3 + R_2$$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & 2 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & -20/9 \end{array}\right] \qquad \left(\frac{-1}{3}\right) R_3$$

The equivalent system of echelon form is

$$x + 2y + z = 2 \qquad \text{(i)}$$

$$y = \frac{5}{3}$$

$$z = \frac{-20}{9}$$

Put value of y and z in eq: (i),

$$x + 2\left(\frac{5}{3}\right) + \left(\frac{-20}{9}\right) = 2$$

$$x + \frac{10}{3} - \frac{20}{9} = 2$$

$$x + \frac{10}{9} = 2$$

$$x = 2 - \frac{10}{9} = \frac{8}{9}$$

Thus $x = \frac{8}{9}$, $y = \frac{5}{3}$; $z = \frac{-20}{9}$

Q.6.(a) If α, β are the roots of the equation $ax^2 + bx + c = 0$, then find the equation whose roots are $\frac{-1}{\alpha^3}, \frac{1}{\beta^3}$. (5)

Ans If α, β are roots of $ax^2 + bx + c = 0$, then

$$\alpha + \beta = \frac{-b}{a} \quad ; \quad \alpha\beta = \frac{c}{a}$$

$$\text{Sum} = S = \frac{-1}{\alpha^3} + \frac{-1}{\beta^3} = \frac{-(\alpha^3 + \beta^3)}{(\alpha\beta)^3}$$

$$= -\left[\frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{(\alpha\beta)^3}\right]$$

$$= -\left[\frac{\left(\frac{-b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)^3}\right]$$

$$= \frac{\frac{-b^3}{a^3} + \frac{3bc}{a^2}}{\left(\frac{c}{a}\right)^3}$$

$$= -\frac{\frac{-b^3 + 3abc}{a^3}}{\frac{c^3}{a^3}} \Rightarrow S = -\left(\frac{-b^3 + 3abc}{c^3}\right)$$

Product of root $= \left(\frac{-1}{\alpha^3}\right)\left(\frac{-1}{\beta^3}\right)$

$$= \frac{1}{(\alpha\beta)^3} = \frac{1}{\left(\frac{c}{a}\right)^3} = \frac{a^3}{c^3}$$

(b) Resolve $\frac{2x^4}{(x-3)(x+2)^2}$ into partial fraction. **(5)**

Ans $\frac{2x^4}{(x-3)(x+2)^2} = \frac{2x^4}{(x-3)(x^2+4+4x)}$

$$= 2x - 2 + \frac{18x^2 + 8x - 24}{x^3 + x^2 - 8x - 12}$$

$$= 2x - 2 + \frac{2(9x^2 + 4x - 12)}{(x-3)(x+2)^2}$$

Let $\frac{9x^2 + 4x - 12}{(x-3)(x+2)^2} = \frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ (i)

Multiply eq. (i) by $(x-3)(x+2)^2$

$9x^2 + 4x - 12 = A(x+2)^2 + B(x-3)(x+2) + C(x-3)$ (ii)

Put $x - 3 = 0 \Rightarrow x = 3$ in eq. (ii),

$9(3)^2 + 4(3) - 12 = A(3+2)^2 + B(3-3)(3-2) + C(3-3)$

$81 + 12 - 12 = (5)^2 A + B(0) + C(0)$

$81 = 25A$

$$\boxed{A = \frac{81}{25}}$$

Put $x + 2 = 0 \Rightarrow x = -2$ in eq. (ii),

$9(-2)^2 + 4(-2) - 12 = A(-2+2)^2 + B(-2-3)(-2+2) + C(-2-3)$

$36 - 8 - 12 = A(0) + B(0) + C(-5)$

$$16 = -5C \quad \Rightarrow \quad \boxed{C = \frac{-16}{5}}$$

Now equation (ii) can be written as

$9x^2 + 4x - 12 = A(x^2 + 4x + 4) + B(x^2 - 3x + 2x - 6) + (x - 3)$

$9x^2 + 4x - 12 = Ax^2 + 4Ax + 4A + Bx^2 - 1Bx - 6B + Cx - 3C$

Comparing the coefficients of x^2, x and constant

$$9 = A + B$$

$$9 = \frac{81}{25} + B$$

$$B = 9 - \frac{81}{25} = \frac{144}{25}$$

Put value of A, B and C in equation (i),

$$\frac{9x^2 + 4x - 12}{(x-3)(x+2)^2} = \frac{81}{25(x-3)} + \frac{144}{25(x+2)} - \frac{16}{5(x+2)^2}$$

Now

$$\frac{2x^4}{(x-3)(x+2)^2} = 2x - 2 + 2$$

$$\left[\frac{81}{25(x-3)} + \frac{144}{25(x+2)} - \frac{16}{5(x+2)^2}\right]$$

Q.7.(a) For what value of n, $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the positive geometric mean (G.M.) between a and b? **(5)**

Ans $\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \sqrt{ab}$

$a^n + b^n = a^{n-½} b^{½} + a^{½} b^{n-1/2}$

$a^n - a^{n-½} b^{½} = a^{½} b^{n-½} - b^n$

$a^{n-½}(a^{1/2} - b^{1/2}) = b^{n-½}(a^{1/2} - b^{1/2})$

$$a^{n-½} = b^{n-½}$$

$$\left(\frac{a}{b}\right)^{n-½} = 1 = \left(\frac{a}{b}\right)^0$$

$$n - \frac{1}{2} = 0 \quad \Rightarrow \quad n = \frac{1}{2}$$

(b) If x is so small that its square and higher powers can be neglected, then show that: (5)

$$\frac{(1-x)^{1/2}(9-4x)^{1/2}}{(8+3x)^{1/3}} \approx \frac{3}{2} - \frac{61}{48}x.$$

Ans L.H.S $= (1-x)^{1/2}(9-4x)^{1/2}(8+3x)^{-1/3}$

$$(1-x)^{1/2} = \left[1 + \frac{1}{2}(-x) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(-x^2)\ldots\right]$$

$$\boxed{(1-x)^{1/2} = 1 - \frac{x}{2}}$$ (Neglect higher powers of x).

$$(9-4x)^{1/2} = 9\left(1 - \frac{4}{9}x\right)^{1/2}$$

$$= (3^2)^{1/2}\left(1 - \frac{4}{9}x\right)^{1/2}$$

$$= 3\left[1 + \frac{1}{2}\left(\frac{-4}{9}x\right) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}\left(\frac{-4x}{9}\right)^2\ldots\right]$$

$$\boxed{(9-4x)^{1/2} = 3\left(1 - \frac{2x}{9}\right)}$$ (Neglect higher powers of x)

$$(8+3x)^{-1/3} = 8\left(1 + \frac{3}{8}x\right)^{-1/3}$$

$$= (2^3)^{-1/3}\left(1 + \frac{3}{8}x\right)^{-1/3}$$

$$= 2^{-1}\left[1 + \left(\frac{-1}{3}\right)\left(\frac{3x}{8}\right) + \frac{\left(\frac{-1}{3}\right)\left(\frac{-1}{3}-1\right)}{2!}\left(\frac{3x}{8}\right)^2 + \ldots\right]$$

$$= \frac{1}{2}\left(1 - \frac{x}{8}\right)$$

$$\boxed{(8+3x)^{-1/3} = \frac{1}{2} - \frac{x}{16}}$$

By putting all values, we have

$$\text{L.H.S} = \left(1 - \frac{x}{2}\right)\left(3 - \frac{2x}{3}\right)\left(\frac{1}{2} - \frac{x}{16}\right)$$

$$= \left(1 - \frac{x}{2}\right)\left[\frac{3}{2} - \frac{3x}{16} - \frac{2x}{6} + \frac{2x^2}{48}\right]$$

$$= \left(1 - \frac{x}{2}\right)\left[\frac{3}{2} - \frac{3x}{16} - \frac{1x}{3} + \frac{1x^2}{24}\right]$$

$$= \left(1 - \frac{x}{2}\right)\left[\frac{3}{2} - \frac{9x + 16x}{48}\right]$$ Neglect x^2

$$= \left(1 - \frac{x}{2}\right)\left(\frac{3}{2} - \frac{25x}{48}\right)$$

$$\frac{3}{2} - \frac{25x}{48} - \frac{3x}{4} + \frac{25x^2}{96}$$

$$\frac{3}{2} - \frac{25x + 36x}{48}$$ Neglect x^2

$$\frac{3}{2} - \frac{61x}{48} \approx \text{R.H.S}$$

Q.8.(a) If $\operatorname{cosec}\theta = \frac{m^2 + 1}{2m}$ and $m > 0$, $\left(0 < \theta < \frac{\pi}{2}\right)$, find the values of the remaining trigonometric ratios. (5)

Ans For Answer see Paper 2018 (Group-I), Q.8.(a).

(b) Prove without using calculator that $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$. (5)

Ans $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{2}\cos 20^\circ \cos 40^\circ \cos 80^\circ$

$$= \frac{1}{4}[2\cos 20^\circ \cos 40^\circ]\cos 80^\circ$$

$$= \frac{1}{4}[\cos 60^\circ + \cos 20^\circ]\cos 80^\circ = \frac{1}{4}[\frac{1}{2} + \cos 20^\circ]\cos 80^\circ$$

$$\frac{1}{8}(1 + 2\cos 20^\circ)\cos 80^\circ$$

$$= \frac{1}{8}[\cos 80^\circ + 2\cos 80^\circ \cos 20^\circ]$$

$$= \frac{1}{8}[\cos 80^\circ + \cos 100^\circ + \cos 60^\circ]$$

$$= \frac{1}{8}[\cos 80^\circ + \cos 100^\circ + \frac{1}{2}]$$

$$= \frac{1}{8}[2\cos\frac{80^\circ + 100^\circ}{2}.\cos\frac{80 - 100}{2} + \frac{1}{2}]$$

$$= \frac{1}{8}[0 + \frac{1}{2}] = \frac{1}{16}$$

Q.9.(a) The sides of a triangle are $x^2 + x + 1$, $2x + 1$ and $x^2 - 1$. Prove that the greatest angle of the triangle is 120°. (5)

Ans We have cosine formula: $\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

Let $a = x^2 + x + 1$, $b = 2x + 1$, $c = x^2 - 1$

Put values of a, b, c in formula,

$$\cos\alpha = \frac{(2x+1)^2 + (x^2-1)^2 - (x^2+x+1)^2}{2(2x+1)(x^2-1)}$$

$$= \frac{4x^2 + 4x + 1 + x^4 - 2x^2 + 1 - (x^4 + x^2 + 1 + 2x^3 + 2x^2 + 2x)}{2(2x^3 - 2x + x^2 - 1)}$$

$$\frac{-2x^3 - x^2 + 2x + 1}{2(2x^3 - 2x + x^2 - 1)} = \frac{-1}{2} = -0.50$$

$$\cos\alpha = -0.50$$

$$\alpha = \cos^{-1}(-0.50)$$

$$\alpha = 120^\circ$$

(b) Prove that $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$. (5)

Ans $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$

$$2\tan^{-1}\frac{1}{3} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{3}$$

$$= \tan^{-1}\frac{3}{4}$$

L.H.S

$$2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4} = \text{R.H.S}$$